A Comparative Analysis of Adaptive Predictive Control Methods Applied in a Heat Exchanger

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***Abstract*—Most industrial processes are nonlinear, which complicates the application of conventional Model-based Predictive Control (MPC) algorithms. Consequently, in this article, the formulations of MPC methods for nonlinear processes represented through polytopic Linear Parameter-Varying models are analysed. The compared methods are adaptive algorithm, synthesised with a prediction model based on a scheduling polytope. At each discrete sampling instant, they determine a model, used for prediction purposes; and optimise the process performances over a finite prediction horizon. These methods are applied to control of a Heat Exchanger system, from which the performance and effectiveness of each technique are discussed. The simulation results are thoroughly analyzed, and the advantages and disadvantages of each strategy are discussed.**

***Index Terms*—model-based predictive control, linear parameter-varying, nonlinear system, heat exchanger**

I. Introduction

It is well known that Model-based Predictive Control (MPC) utilize a model to predict future system behavior and determine the optimal control action [1]. For this reason, the predictive model assumes a crucial role in MPC formulations. While Linear Time-Invariant (LTI) models are suitable for systems operating close to their nominal point, as it allows for the use of well-established linear control tools for analysis and design [2], nonlinear systems are often represented using the Linear Parameter-Varying (LPV) modeling approach. The LPV framework enables more accurate representation of system dynamics, considering the varying nature of the system and its dependence on operating conditions. Therefore, by incorporating the LPV framework, control strategies can effectively handle nonlinearity and achieve improved performance.

The control problem for systems with parametric uncertainty has been addressed in many works in the specialized literature. Several works have proposed predictive control approaches using infinite prediction horizons and fixed linear control laws based on the state variable. Linear Matrix Inequality (LMI) techniques are often employed to solve the optimization problem in these cases, as seen in [3]–[5], and others. Meanwhile, in [6], [7], among others, an explicit MPC based on parametric programming is proposed for the same purpose. In these works, the optimal inputs are computed offline as a piecewise affine function of the states and stored in a lookup table. Then, only the table needs to be evaluated, allowing for the application of MPC to the system.

However, this article focuses on analyzing MPC control methods that consider the properties and characteristics of MPC for regulation found in the literature ([8]–[10], among others), while also taking into account a nonlinear system represented using the LPV approach as a predictive model. This includes the MPC controllers presented in [11] and [12].

This paper is organised as follows. In Section II, the model representation of a system is provided. In Section III a briefly explanation of the MPC strategies is introduced. Subsequently, in Section IV a numerical simulations is provided to show the performance of the proposed approach. Finally, concluding remarks are made in the last section.

II. Model statement

A. LPV Model

The uncertain system dynamics can be described by a discrete state-space model with polytopic uncertainty using the LPV approach:

$$\begin{aligned} x(k+1) &= A(\rho(k))x(k) + B(\rho(k))u(k)\,, \\ y(k) &= C(\rho(k))x(k) + D(\rho(k))u(k)\,, \\ x(0) &= x_0\,, \end{aligned} \tag{1}$$

where the parameter $\rho(k)$ is an scheduling parameter that takes into account the minimum and maximum values that each parameter can assume within the operating range under consideration. Its purpose is to incorporate the maximum uncertainty associated with these parameters, and it is determined by a nonlinear function. It is important to highlight that the considered uncertainty is always contained within a bounded and closed set.

Subsequently, it is possible to incorporate the dynamics of a nonlinear system within a convex polytope with n_m vertices through the Parametrized Jacobian Linearization (PJL) technique [13], [14]. These vertices are obtained based on the maximum uncertainty, although intermediate values in the system parameters can be considered for improved performance. In this way, the system is contained within a set of LTI models at its n_m vertices:

$$[A(\rho(k)), B(\rho(k)), C(\rho(k)), D(\rho(k))] \in \Omega\,,$$

which is represented as:

$$\Omega = \mathrm{Co}\{[A_1, B_1, C_1, D_1], [A_2, B_2, C_2, D_2], \ldots, [A_{n_m}, B_{n_m}, C_{n_m}, D_{n_m}]\},$$

where $\mathrm{Co}\{\cdot\}$ denotes a convex hull, and $[A_j, B_j, C_j, D_j]$ are the matrices of each vertex LTI model of Ω. The number of vertex models is given by $n_m = n_l^{n_p}$, where n_p represents the number of adjustment parameters and n_l represents the number of linearization points per parameter.

Utilizing interpolation techniques on the vertex LTI models, it becomes feasible to derive a predictive model that encompasses not only the vertices but also the models contained within Ω:

$$\begin{aligned} A(\rho(k)) &= \sum_{j=1}^{n_m} \mu_j(\rho(k)) A_j\,, \quad B(\rho(k)) = \sum_{j=1}^{n_m} \mu_j(\rho(k)) B_j \\ C(\rho(k)) &= \sum_{j=1}^{n_m} \mu_j(\rho(k)) C_j\,, \quad D(\rho(k)) = \sum_{j=1}^{n_m} \mu_j(\rho(k)) D_j \end{aligned} \tag{2}$$

where it must be ensured that:

$$\sum_{j=1}^{n_m} \mu_j(\rho(k)) = 1\,, \quad 0 \le \mu_j(\rho(k)) \le 1\,, \quad \forall j \in \mathbb{Z}_{1:n_m}\,. \tag{3}$$

The weighting variable $\mu(\cdot) \in \mathbb{R}^{n_m}$ represents the weight of each vertex LTI model relative to the uncertain model. If this variable is known at each time instant, the convex sum defined by Eqs. (2) and (3) can be used to determine the LTI model that represents the system at that instant.

However, $\mu(\cdot)$ depends on $\rho(k)$, which often exhibits nonlinear and unknown evolution. Thus, there are two main alternatives for designing an MPC controller under the LPV approach [15]. One option is to adopt a robust design, assuming that the adjustment parameters are unknown throughout the prediction horizon. These methods are therefore more conservative and typically rely on min-max procedures or offline formulations.

Another alternative is to determine a possible trajectory of $\rho(k)$ over future steps. In the case of nonlinear systems, it should be noted that the resulting solution may deviate slightly from the nonlinear optima, but good performances are achieved with low computational effort. In many cases, comparable results, meaning close to the optimum, can be obtained.

B. Heat Exchanger

In practice, every chemical process involves the production or absorption of energy in the form of heat. Heat Exchangers (HE) are widely used in the process industry to transfer heat from a hot fluid to a cold fluid through a solid wall [16].

The process model of a HE presented in Fig. 1 is obtained based on the assumptions and conditions presented in [17]. Then, the balance equations that describe the lumped-parameter nonlinear model are:

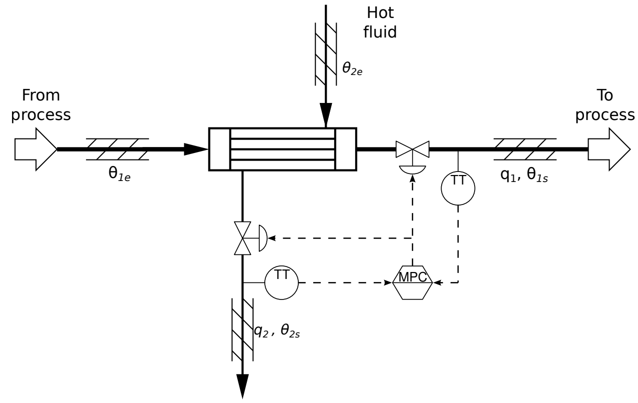


Fig. 1: Heat exchanger process diagram.

$$\begin{aligned} \frac{d\theta_{1s}(t)}{dt} &= \frac{q_1 \rho_1 C_{p_1}(\theta_{1e} - \theta_{1s}(t)) - Ah_1(\theta_{1_s}(t) - \theta_p(t))}{\rho_1 C_{p_1} V_1} \\ \frac{d\theta_{2s}(t)}{dt} &= \frac{q_2 \rho_2 C_{p_2}(\theta_{2_e} - \theta_{2s}(t)) + Ah_2(\theta_p(t) - \theta_{2_s}(t))}{\rho_2 C_{p_2} V_2} \\ \frac{d\theta_p(t)}{dt} &= \frac{Ah_1(\theta_{1s}(t) - \theta_p(t)) - Ah_2(\theta_p(t) - \theta_{2s}(t))}{\rho_p C_{p_p} V_p} \end{aligned} \tag{4}$$

where θ_{1s} (outlet temperature of fluid 1), θ_{2s} (outlet temperature of fluid 2), and θ_p (wall temperature) correspond to the system states, and q_1 (process fluid flow rate) and q_2 (heating fluid flow rate) are the control variables.

The corresponding physical and operational parameters are summarized in Table I.

TABLE I: Parameters of the HE model.

Parameter	Description	Value
ρ_1, ρ_2	Density of fluid 1 and 2	$1\,\mathrm{kg\,L^{-1}}$
ρ_p	Density of the wall	$7.874\,\mathrm{kg\,L^{-1}}$
C_{p_1}, C_{p_2}	Specific heat capacity of fluid 1 and 2	$1000\,\mathrm{cal\,kg^{-1}\,K^{-1}}$
C_{p_p}	Specific heat capacity of the wall	$1075.53\,\mathrm{cal\,kg^{-1}\,K^{-1}}$
A	Exchange area	$0.881\,\mathrm{m^2}$
h_1	Heat transfer of fluid 1	$32\,374\,\mathrm{cal\,min^{-1}\,K^{-1}\,m^{-2}}$
h_2	Heat transfer of fluid 2	$14\,716.6667\,\mathrm{cal\,min^{-1}\,K^{-1}\,m^{-2}}$
V_1	Tube volume	$16\,\mathrm{L}$
V_2	Shell volume	$2.11\,\mathrm{L}$
V_p	Wall volume	$1.19\,\mathrm{L}$
θ_{1e}	Inlet temperature of fluid 1	$450\,\mathrm{K}$
θ_{2e}	Inlet temperature of fluid 2	$900\,\mathrm{K}$

By applying the PJL technique, it is possible to rewrite the system (4) as an LTI model around the j-th operating point $x_j = \{\theta_{1s_j},\ \theta_{2s_j},\ \theta_{p_j}\}$,

$$\begin{aligned} A_j &= \begin{bmatrix} -\frac{Ah_1 + q_{1_j}\rho_1 C_{p_1}}{V_1 \rho_1 C_{p_1}} & 0 & \frac{Ah_1}{V_1 \rho_1 C_{p_1}} \\ 0 & -\frac{Ah_2 + q_{2_j}\rho_2 C_{p_2}}{V_2 \rho_2 C_{p_2}} & \frac{Ah_2}{V_2 \rho_2 C_{p_2}} \\ \frac{Ah_1}{V_p \rho_p C_{p_p}} & \frac{Ah_2}{V_p \rho_p C_{p_p}} & -\frac{Ah_1 + Ah_2}{V_p \rho_p C_{p_p}} \end{bmatrix}, \\ B_j &= \begin{bmatrix} \frac{\theta_{1_e} - \theta_{1s_j}}{V_1} & 0 \\ 0 & \frac{\theta_{2_e} - \theta_{2s_j}}{V_2} \\ 0 & 0 \end{bmatrix}, \end{aligned} \tag{5}$$

meanwhile, the matrix C is the identity matrix and the matrix D is zero, with appropriate dimensions.

III. Adaptive MPC methods

Based on the previous sections, in Adaptive MPC (AMPC) methods using the LPV formalism, it is necessary to know, calculate, or approximate the weighting parameter $\mu \in \mathbb{R}^{n_m}$. Therefore, μ is considered as a decision variable and the conditions given in Eqs. (2) and (3) become constraints of the optimization problem.

Thus, depending on how μ is determined, different formulations arise.

A. Single-Stage Adaptive MPC

In this approach based on [12], it is considered a single optimization problem over a prediction horizon of N steps, where a predictive model, as indicated in Eq. (1), is used. Additionally, the states and the control variable are subject to constraints, given by:

$$x(k) \in \mathcal{X} \quad \text{and} \quad u(k) \in \mathcal{U}\,, \tag{6}$$

where $\mathcal{X}$ and $\mathcal{U}$ are convex and compact subsets of $\mathbb{R}^{n_x}$ and $\mathbb{R}^{n_u}$, respectively. Furthermore, the objective function is defined as:

$$\begin{aligned} V_N(x;\mathbf{u},\mu) = \sum_{k=0}^{N-1} \|x(k)\|_Q^2 + \|u(k)\|_R^2 + \|x(N)\|_P^2 \\ + \|\mu(N) - \mu_s\|_{Q_\mu}^2\,, \end{aligned} \tag{7}$$

with $Q \in \mathbb{R}^{n_x \times n_x}$ and $Q_\mu \in \mathbb{R}^{n_m \times n_m}$ positive semidefinite, and $R \in \mathbb{R}^{n_u \times n_u}$ positive definite. μ_s specifies the combination of vertex models that best represent the system at the established operating point. The decision variables correspond to the control sequence $\mathbf{u}$ and the model sequence obtained through μ. Thus, at each sampling instant, this technique seeks the best sequences of LTI prediction models for the next N steps.

In this way, the controller is derived from the solution of the optimization problem:

$$\begin{aligned} \min_{\mathbf{u},\mu} \quad & V_N(x_0;\mathbf{u},\mu) \\ \textbf{s.t.} \quad & x(k+1) = A(\rho(k))x(k) + B(\rho(k))u(k)\,, \\ & x(0) = x_0 \\ & A(\rho(k)) = \sum_{j=1}^{n_m} \mu_j(k) A_j\,, \\ & B(\rho(k)) = \sum_{j=1}^{n_m} \mu_j(k) B_j\,, \\ & \sum_{j=1}^{n_m} \mu_j(k) = 1\,, \qquad 0 \le \mu_j(k) \le 1\,, \\ & x(k) \in \mathcal{X}\,, \qquad u(k) \in \mathcal{U}\,, \\ & x(N) \in \mathcal{X}_f^a\,. \end{aligned} \tag{8}$$

where $k \in \mathbb{Z}_{0:N-1}$ and $\mathcal{X}_f^a$ is a robust invariant set for the LPV model, meaning that it satisfies the invariance condition for every model belonging to Ω.

By considering μ as a varying variable throughout the prediction horizon, results close to the optimum can be achieved. However, this leads to a nonlinear optimization problem. If computational burden needs to be reduced, a constant μ can be considered within the horizon, which clearly leads to suboptimal results. In this case, a constant weighting variable can be obtained by optimizing the model in the control stage or in a previous estimation stage, and then used for control as discussed in the following section.

B. Two-Stages Adaptive MPC

Given the high computational cost associated with considering a sequence of models for prediction, this section presents a two-stage design procedure for AMPC based on [11]. In this approach, a constant LTI prediction model is used throughout the prediction horizon, which is obtained previously using the LPV model indicated in Eq. (1). The physical constraints given in Eq. (6) are also taken into account.

In this way, at each sampling time, two Quadratic Programming (QP) problems are solved: the first QP considers a backward horizon to find the virtual adjustment variable between model-system, denoted as μ. This variable defines the LTI model that best describes the system at that instant, based on data from the previous N_e steps and considering the vertices of the polytopic model. Then, the second QP uses this LTI model as the prediction model to optimize performance over a future horizon.

The first problem is used to find a constant vector $\mu \in \mathbb{R}^{n_m}$ that optimally adjusts the LPV model with the past real data. In fact, this procedure minimizes the discrepancy between the model and the data with respect to μ and the variance of μ (ν_μ) over the backward estimation horizon (N_e) at each sampling time.

$$\begin{aligned} \min_{\mu} \quad & \sum_{k=k_0+N_e-1}^{k_0} e(k)^T Q_e e(k) + \nu_\mu^T Q_\nu \nu_\mu \\ \textbf{s.t.} \quad & e(k+1) = x(k+1) - (Ax(k) + Bu(k))\,, \\ & A = \sum_{j=1}^{n_m} \mu_j A_j \quad \text{and} \quad B = \sum_{j=1}^{n_m} \mu_j B_j\,, \\ & \sum_{j=1}^{n_m} \mu_j = 1\,, \qquad 0 \le \mu_j \le 1\,, \\ & \mu = \mu(k_0 - 1) + \nu_\mu\,, \end{aligned} \tag{9}$$

with $k \in \mathbb{Z}_{k_0-N_e:k_0-1}$, $j \in \mathbb{Z}_{1:n_m}$.

In this way, the MPC problem is formulated with the following cost function, considering that μ represents the value obtained from the problem (9):

$$V_N(x_0,\mu;\mathbf{u}) = \sum_{k=0}^{N-1} \|x(k)\|_Q^2 + \|u(k)\|_R^2 + \|x(N)\|_P^2\,, \tag{10}$$

where N is the prediction horizon.

The optimal control sequence is obtained by solving the following optimization problem:

$$\begin{aligned} \min_{\mathbf{u}} \quad & V_N(x_0,\mu;\mathbf{u}) \\ \textbf{s.t.} \quad & x(k+1) = Ax(k) + Bu(k)\,, \forall k \in \mathbb{Z}_{0:N-1}\,, \\ & x(0) = x_0\,, \\ & A = \sum_{j=1}^{n_m} \mu_j A_j \quad \text{and} \quad B = \sum_{j=1}^{n_m} \mu_j B_j\,, \\ & x(k) \in \mathcal{X}\,, \qquad u(k) \in \mathcal{U}\,, \\ & x(N) \in \mathcal{X}_f^a\,, \end{aligned} \tag{11}$$

where $\mathcal{X}_f^a$ is a robust invariant terminal set, obtained using the same approach as for the controller presented in the Section III-A. The calculation of this set is detailed in Section III-C.

C. Stability and feasibility analysis

To guarantee asymptotic stability and recursive feasibility, it is necessary to define the terminal cost as a control Lyapunov function and the terminal set as a control robust positive invariant set for all models belonging to Ω. In this way, to define P and $\mathcal{X}_f^a$, a polytopic LQR without constraints is considered as a local controller with the model given in Eq. (1). Thus, a $P \succ 0$ and a control law $u(k) = \kappa x(k)$ for the LQR need to be found that satisfy

$$[A(\rho(k)) - B(\rho(k))\kappa]^T P [A(\rho(k)) - B(\rho(k))\kappa] + Q + \kappa^T R \kappa - P \leq 0\,. \tag{12}$$

Therefore, P is determined by solving the following problem:

$$\begin{aligned} &\min_{\gamma, L, Y} \ \gamma \\ &\textbf{s.t. } Y \succ 0\,, \\ &\begin{pmatrix} Y & Y A_j^T + L^T B_j^T & Y Q^{1/2} & L^T R^{1/2} \\ A_j Y + B_j L & Y & 0 & 0 \\ Q^{1/2} Y & 0 & \gamma I & 0 \\ R^{1/2} L & 0 & 0 & \gamma I \end{pmatrix} \succ 0\,, \end{aligned} \tag{13}$$

for all $j \in \mathbb{Z}_{1:n_m}$. Then, it is ensured that the terminal cost is decreasing for all $x \in \mathcal{X}_f^a$. As a result, a region defined by a robust invariant set $\mathcal{X}_f^a$ needs to be defined, where the decrease of the terminal cost is satisfied.

As mentioned in [18], [19], for the regulation purpose, the largest invariant ellipsoidal set is considered as the terminal constraint.

In this way, it is possible to determine a terminal invariant ellipsoid for the terminal state $x(N)$, centered at the origin. The ellipsoid is given by:

$$\mathcal{X}_f^a = \left\{ x(N) : x(N)^T W x(N) \leq 1 \right\}\,. \tag{14}$$

where this terminal set is a subset of the terminal cost $x(N)^T P x(N)$.

Therefore, to find the largest invariant terminal set $\mathcal{X}_f^a$ under the control law $u(k) = \kappa x(k)$ for all k with admissible input, it is possible to formulate a second maximization LMI problem:

$$\begin{aligned} &\max_{Z} \ \log \det(Z) \\ &\textbf{s.t. } Z \succ 0\,, \\ &\begin{pmatrix} Z & Z(A_j + B_j\kappa)^T \\ (A_j + B_j\kappa) Z & Z \end{pmatrix} \succ 0\,,\ \forall j \in \mathbb{Z}_{1:n_m}\,, \\ &Z \leq \mathcal{X}_{max}\,, \\ &\kappa_s Z \kappa_s^T \leq u_{s_{max}}^2\,, \end{aligned} \tag{15}$$

where $W = Z^{-1}$, $s \in \mathbb{Z}_{1:n_u}$ denotes each component of the vector $u(k)$, and κ_s are the rows of the matrix $\kappa = LY^{-1}$ determined by the LMI problem presented in Eq. (13).

Thereby, a terminal cost and an invariant terminal set are obtained for the n_m vertex LTI models, and due to the convexity of Ω, these properties extend to every model belonging to the model polytope [12].

IV. Simulation results

The following results comprise the constrained regulation of the outlet temperature x_1, despite disturbances in the output variables of 0.01 K. In addition, constraints are considered for the states $x_1(t) = \theta_{1s}(t) \in [495, 500]$ K, $x_2(t) = \theta_{2s}(t) \in [650, 710]$ K and $x_3(t) = \theta_p(t) \in [530, 580]$ K; and for the control variables $u_1(t) = q_1(t) \in [90, 110]$ $\mathrm{L\,min^{-1}}$ and $u_2(t) = q_2(t) \in [7, 9]$ $\mathrm{L\,min^{-1}}$.

To establish the LPV model (5), two adjustment parameters, $\rho(t) := \begin{bmatrix}\theta_{1s}(t) & \theta_{2s}(t)\end{bmatrix}$, are defined, with two linearization points per parameter, $n_l = 2$. Therefore, four vertex LTI models are fixed. The linearization points for these models are set to cover the possible operating range and are obtained from the combination of values shown in Table II.

TABLE II: LPV model of a HE.

	θ_{1s}	θ_{2s}
Model 1	495 K	680 K
Model 2	500 K	680 K
Model 3	495 K	710 K
Model 4	500 K	710 K

At the same time, the sampling period is set to $T = 0.05$ min, the prediction horizon $N = 8$ steps is chosen based on the observation of the system's slow dynamics and its ability to react to changes, the initial state is $x_0 = [495; 710; 562.1913]$ K, the MPC weight matrices, in accordance with traditional methods, are $Q = \mathbb{I}$ and $R = diag\{1; 0.1\}$ in relation to response time and control variable amplitudes, the estimator weight matrices are $Q_e = 1 \times 10^6 \mathbb{I}$ and $Q_\nu = \mathbb{I}$, and the AMPC model weight matrix is $Q_\mu = 1 \times 10^6 \mathbb{I}$. The operating point is fixed at $x_s = [498; 680; 554.8782]$ K. It is important to note that all the implemented MPC algorithms use the same set of weighting matrices and prediction horizon. In the case of two-stage AMPC, identify as MHE-MPC, the estimation horizon is set to be equal to the prediction horizon.

The control methodologies proposed in Section III are compared among themselves, but also with a linear MPC (referred to as LTI-MPC), which solves the QP problem given in Eq. (11) by setting all the weighting variables as $\mu_j = 1/n_m$, which represents an LTI model for the analyzed systems, considering the n_m vertices of the polytope Ω.

The evolution of the system states is presented in Fig. 2, which converges to the set operating point. Additionally, the figure shows the state constraint set (in red) and the terminal invariant set (in green). It is noteworthy that the terminal invariant set is feasible as it is entirely contained within the constraint set. The figure depicts the state evolution using the AMPC (III-A) in blue line, the MHE-MPC (III-B) in red line, and the LTI-MPC in green line. The response of each controller is analyzed in detail in the subsequent figures.

Figure 3a shows the temporal evolution of the states when the control variables (Fig. 3c) obtained by each of the analyzed controllers are applied. It can be observed that during the first 1.25 min, the system evolves to reach the operating point, and then the states remain in a steady-state. Figure 3b provides a

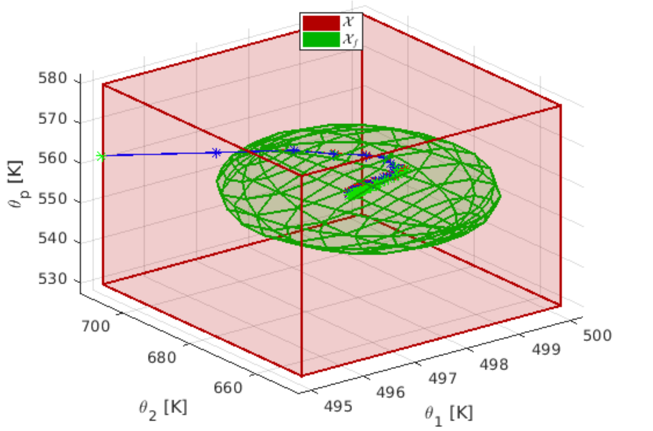


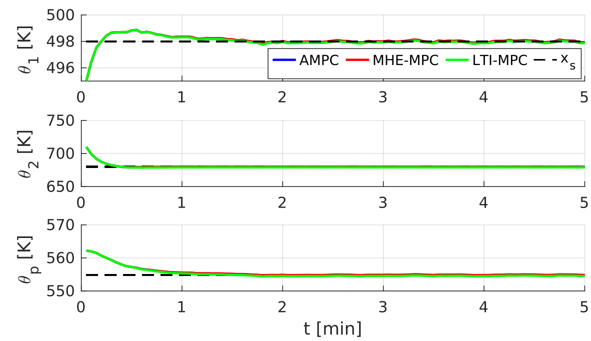
Fig. 2: Evolution of the HE system states for regulation to x_s.

zoomed-in view of the temporal evolution of the states, where the effect of noise in the output variables and the steady-state error produced by the LTI-MPC can be noticed.

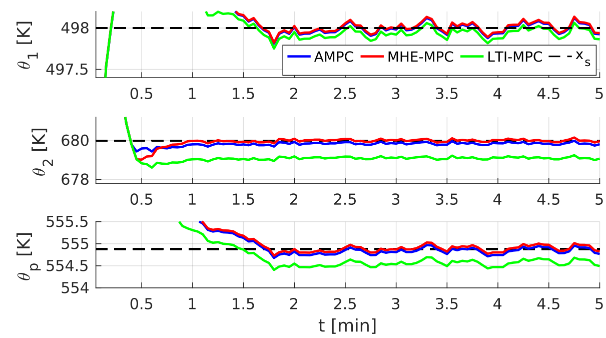
As discussed in this work, the adaptive MPC techniques aim to adapt the prediction model through the weighting variables μ_j. In relation to this, Fig. 4 shows the evolution of the weighting variables over time. In Fig. 4a, the membership of each vertex model to the prediction model is presented when using the AMPC. It can be observed that, based on the specified operating range and the nonlinearities of the system, a linear model is sufficient to address the control problem, this demonstrates that this controller is a generalization of the LTI-MPC formulation. In this case, the optimal prediction model is composed of 90.25% of model 2 and 9.75% of model 4. It is important to highlight that, although in this example a linear model allows for an approximate prediction of the system's behavior, the adaptability feature has the advantage that it can reformulate the controller without the need for obtaining new models when there is a change in the operating point, which is often not a trivial task.

On the other hand, Fig. 4b illustrates the evolution of the weighting variables obtained by the estimation stage of the MHE-MPC. It can be observed in the figure the change of prediction model that occurs over time, where initially model 3 is used, then models within Ω are employed, and finally model 2 is applied. So it is very close to the model obtained by the AMPC.

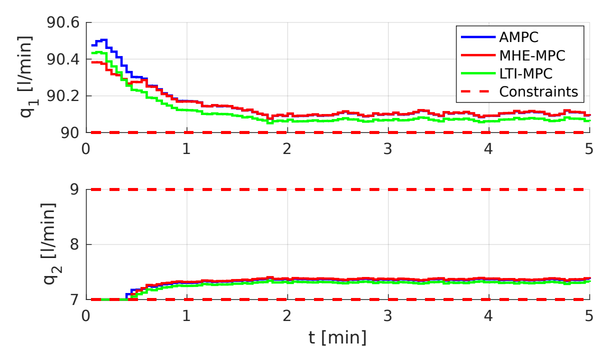
To highlight the results, Table III shows the Integral Absolute Error (IAE) and Integral Time-weighted Absolute Error (ITAE) indices for the transient regime, as well as the steady-state results (with respect to x_1). Additionally, Table III also presents the Total Variance (TV) index for the three methods. Higher values for the TV index indicate more variation in control throughout the simulation. Therefore, values closer to zero indicate better (smoother) control strategies in terms of actuator usage. The analysis of the TV index is crucial from a practical point of view, as it implies that the system's actuators



(a) System states.



(b) Zoomed-in view of system states.



(c) Control variables.

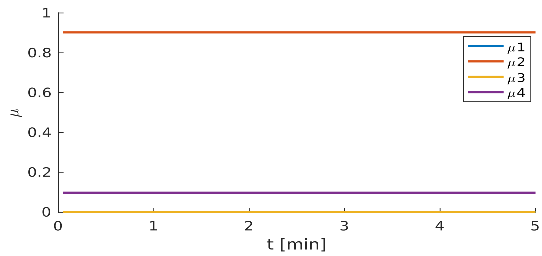
Fig. 3: Temporal evolution of the HE system variables.

will have a longer lifespan. The table expresses the best value for each index as a reference, and the remaining values are expressed in terms of the percentage increase compared to the best value.

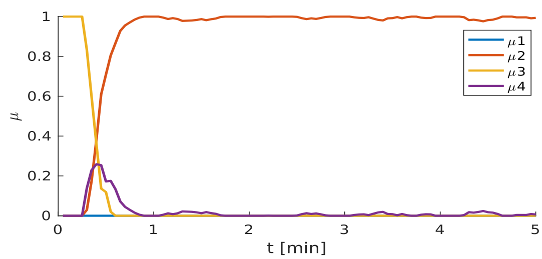
TABLE III: Performance indices in a HE.

	IAE trans.	ITAE trans.	IAE ss.	ITAE ss.	TV
AMPC	$+2,534\%$	$+5,862\%$	$\mathbf{0,22433}$	$\mathbf{0,64506}$	$+0,390\%$
MHE-MPC	$+2,956\%$	$+6,101\%$	$+1,306\%$	$+2,845\%$	$+0,353\%$
LTI-MPC	$\mathbf{0,6831}$	$\mathbf{0,33452}$	$+29,786\%$	$+30,735\%$	$\mathbf{38.0155}$

In addition, Table IV presents the Online Computational Effort (OCE) in terms of maximum, average and minimum elapsed computational time as a percentage of the sampling time.



(a) AMPC.



(b) MHE-MPC.

Fig. 4: Evolution of the membership variables for obtaining the predictive model.

TABLE IV: Online computational effort in a HE.

	minimum	average	maximum
AMPC	14.4744%	24.428%	55.3617%
MHE-MPC	0.6338%	0.7092%	3.1255%
LTI-MPC	**0.4183%**	**0.45863%**	**0.51357%**

As a result, the quantitative results presented in Tables III and IV, as well as the observations from the figures, demonstrate that for the given operating range, a linear prediction model is adequate. In this regard, the performance indices for reference tracking and TV indicate a slightly better performance for the LTI-MPC technique. However, the adaptive techniques reduce the steady-state error and improve disturbance rejection, as reflected in the corresponding performance indices, where a significant difference is observed. On the other hand, the LTI-MPC requires less OCE. However, all methods can be implemented for real-time purposes, as the maximum elapsed time is smaller than the sampling time, which can be considered an acceptable computation time. Nonetheless, the AMPC technique exhibits a significantly higher computational effort, which may pose challenges for its implementation in systems with a small sampling time. However, it is reasonable to use AMPC in real-time applications with a sampling time in the range of a few seconds.

V. Conclusion

In conclusion, this work analyzed two adaptive predictive control strategies for regulation, applicable to nonlinear systems represented using the LPV model approach. These techniques extend the formulation of traditional MPC for regulation purpose to a multi-model representation of the plant.

To achieve this, the control of a HE was addressed. A bounded operating range was considered, where an MPC with an LTI prediction model was able to control the nonlinear system. Under these conditions, it was observed that the three analyzed techniques exhibited similar performance during the transient regime. However, the adaptive techniques showed improvements in disturbance rejection and offset reduction. It is also worth noting that the adaptability of these techniques provides the advantage that, in the case of a change in the operating point, there is no need to obtain new models when reformulating the controller, which can be a non-trivial task for nonlinear industrial systems.

Thus, the analyzed adaptive MPC methods ensured the regulation of nonlinear systems represented by a polytopic model used for prediction, without the need to evaluate the nonlinear differential equations describing the dynamics of each system online.

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